

Reg. No. : 

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**Question Paper Code : 91354**

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering ))

(Regulations 2021)

Time : Three hours

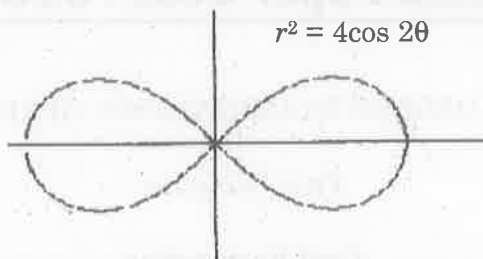
Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Can a real matrix have complex eigenvalues? Justify the answer.
2. What is diagonalization? Why does the matrix  $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  not diagonalizable?
3. Find the value of  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$ .
4. If  $x = a(\cos t + t \sin t)$ ;  $y = a(\sin t - t \cos t)$ , find  $\frac{d^2 y}{dx^2}$ .
5. Find the second order partial derivatives of  $z = x^3 + y^3 - 3axy$ .
6. Using Jacobians, verify  $\frac{\partial(x, y)}{\partial(r, \theta)} = r$ , where  $x = r \cos \theta, y = r \sin \theta$ .
7. Obtain the value of the definite integral  $\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$ .
8. For what values of  $p$  in the integral  $\int_1^{\infty} \frac{1}{x^p} dx$  convergent?

9. Evaluate  $\iint_R \frac{\sin x}{x} dA$  where  $R$  is the triangle in the  $xy$ -plane bounded by the  $x$ -axis, the line  $y = x$  and the line  $x = 1$ .
10. Find the area enclosed by the lemniscate (shown figure below)  $r^2 = 4 \cos 2\theta$ .



PART B — (5 × 16 = 80 marks)

11. (a) State Cayley-Hamilton theorem. Verify the Cayley theorem for the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$  and hence find the inverse of  $A$ . Also find  $A^4$ .

Or

- (b) Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$  to the canonical form and specify the matrix of transformation.

12. (a) (i) Find  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ . (8)

- (ii) Find an equation of the tangent and normal lines to the given curve at specified point  $f(x) = \frac{x^2 - 1}{x^2 + x + 1}$ , at  $(1, 0)$  (8)

Or

- (b) (i) Find the maximum and minimum values of  $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$  in the interval  $(0, 2)$ . (8)

- (ii) Water runs into a conical tank at the rate of 9 ft<sup>3</sup>/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep? (8)

13. (a) (i) Using Euler theorem, prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$  where

$$\log u = \frac{x^3 + y^3}{3x + 4y}. \quad (8)$$

- (ii) Apply Taylor's formula to find the cubic approximation of  $f(x) = \log_e x$  in about  $x = 1$ . Hence evaluate  $\log_e 1.1$  correct to four decimal places. (8)

Or

- (b) (i) Find the absolute maximum and minimum values of

$$f(x, y) = 2 + 2x + 4y - x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines  $x = 0$ ,  $y = 0$  and  $y = 9 - x$ . (8)

- (ii) Find the maximum and minimum values of the function  $f(x, y) = 3x + 4y$  on the circle  $x^2 + y^2 = 1$  using Lagrange's multipliers method. (8)

14. (a) Evaluate the following integrals using appropriate substitutions:

(i)  $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$  (8)

(ii)  $\int_0^\pi \frac{x \sin^3 x}{1 + \cos^2 x} dx$  (8)

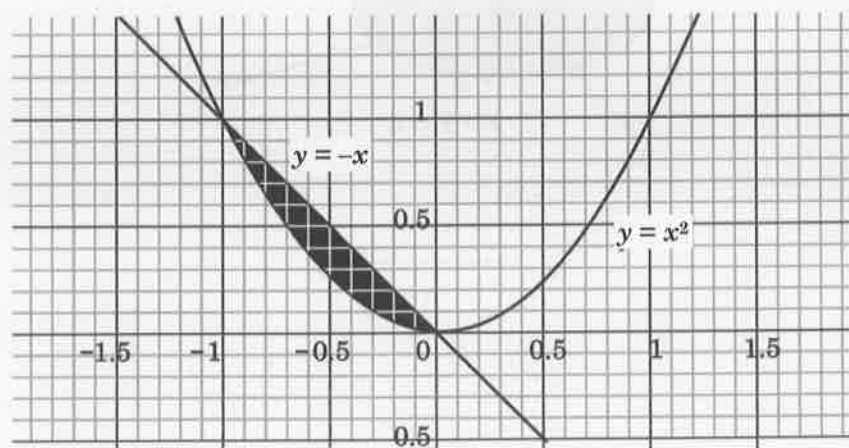
Or

- (b) Use partial fractions to evaluate the following integrals :

(i)  $\int \frac{x^2 + 2x + 1}{(x-1)(x-2)(x-3)} dx$  (8)

(ii)  $\int \frac{x^2}{(x-1)(x^2 + 2x + 1)} dx$  (8)

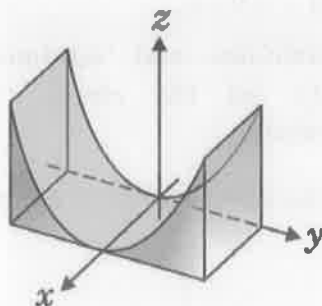
15. (a) (i) Find the area of the shaded region given in the Figure below by double integration. (8)



- (ii) Sketch the region of integration for the integral  $\int_0^2 \int_{x^2}^{2x} (4x+2) dy dx$  and write an equivalent double integral with the order of integration reversed. Also evaluate the integral using the order of integration reversed. (8)

Or

- (b) (i) Find the volumes of the region between the cylinder  $z = y^2$  and the  $xy$ -plane that is bounded by the planes  $x = 0$ ,  $x = 1$ ,  $y = -1$  and  $y = 1$ . (8)



- (ii) Find the center of mass of a solid of constant density bounded below by the disk  $R: x^2 + y^2 \leq 4$  in the plane  $z = 0$  and above by the paraboloid  $z = 4 - x^2 - y^2$  (shown in figure). (8)

